

MARKSCHEME

May 2001

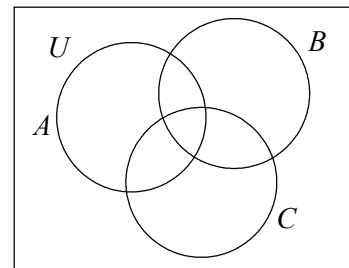
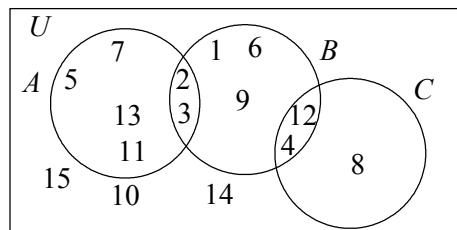
MATHEMATICAL STUDIES

Standard Level

Paper 2

1. (a) (i) $A = \{2, 3, 5, 7, 11, 13\}$ (A1)
- (ii) $B = \{1, 2, 3, 4, 6, 9, 12\}$ (A1)
- (iii) $C = \{4, 8, 12\}$ (A1)
- (iv) $A \cap B \cap C = \emptyset$ (A1)
- [4 marks]

(b) (i)(ii)



Also accept with elements correctly placed

(A2)(A2)

Note: Allow fit from part (a).
Award (A1) for rectangle labelled, (A1) for circles in correct places.
Award (A1) for correct elements in intersections, award (A1) for the rest of the elements correctly placed. IF the wrong Universal set is used, penalise only one mark.

[4 marks]

(c) **Note:** Award follow through from part (b) only.

- (i) $A \cap (B \cup C) = \{2, 3\}$ (A1)
- (ii) $(A \cap B)' = \{1, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$ (A1)

Note: Award follow through from part (c) (ii).

- (iii) $(A \cap B)' \cap C = \{4, 8, 12\}$ (A1)

[3 marks]

continued...

Question 1 continued

(d) **Note:** Award follow through from either part (a) or part (b).

(i) $\frac{6}{15} = \frac{2}{5}$ or 0.4 or 40% **(A1)**

(ii) $\frac{4}{15}$ or 0.267 or 26.7% (3 s.f.) **(A1)**

Note: In parts (iii) and (iv), award **(A1)** for the correct numerator, **(A1)** for the correct denominator.

(iii) $\frac{6}{15}$ or $\frac{2}{5}$ or 0.4 or 40% **(A2)**

(iv) $\frac{2}{7}$ or 0.286 or 28.6% (3 s.f.) **(A2)**

[6 marks]

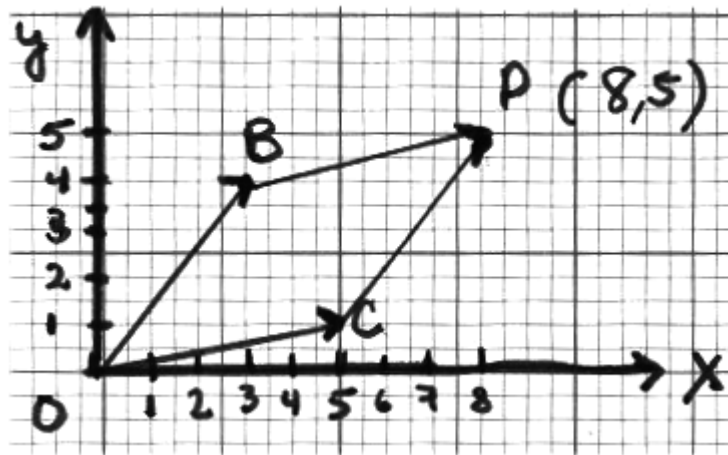
Total [17 marks]

2. (a) (i) $\vec{OB} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ (A1)

(ii) $\vec{OC} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$ (A1)

[2 marks]

(b) (i)



(A2)

Note: If P is plotted incorrectly, but moved correctly according to graph, (A0)(A1) follow through.

Note: The vector for \vec{CP} is not necessary – only point P and its coordinates.

Note: Award (A1) for the point P correctly plotted, (A1) for the correct coordinates (8, 5).

(ii) Parallelogram (A1)

[3 marks]

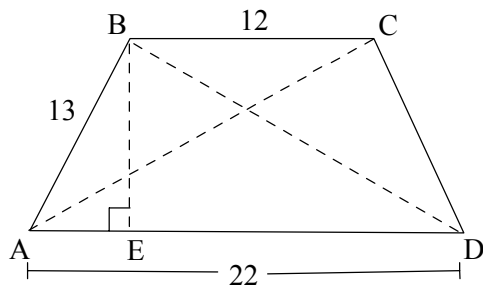
(c) $\left| \vec{OP} \right| = \sqrt{(-8)^2 + 5^2}$ (M1)

$= \sqrt{89}$ or 9.43 (3 s.f.) (A1)

[2 marks]

Total [7 marks]

3.



- (a) $22 - 12 = 10$ (R1)
 Therefore, $AE = \frac{10}{2} = 5$ (R1)(AG)
 Also allow $12 + 2(5) = 22$. (R2)
[2 marks]

- (b) $13^2 = 5^2 + BE^2$ (M1)
 $BE = \sqrt{169 - 25}$
 $= 12 \text{ cm}$ (A1)
 Also allow just an answer 12 (Pythagorean triple) (C2)
[2 marks]

- (c) (i) $\tan \hat{BAE} = \frac{12}{5}$ (accept any other correct ratio) (M1)
 $= 2.4$
 $\hat{BAE} = 67.4^\circ$ (3 s.f.) (A1)
 (ii) $\hat{BCD} = 180 - 67.4$ (A1)
 $= 113^\circ$ (3 s.f.)
[3 marks]

- (d) $CA^2 = BD^2 = 13^2 + 22^2 - 2(13)(22)\cos 67.4^\circ$ (M1)
 $= 433.183$ (M1)
 $CA = 20.8$ (3 s.f.) (A1)

OR

- $ED = 17$ (M1)
 $CA^2 = BD^2 = 12^2 + 17^2 = 433$ (M1)
 Therefore, $CA = 20.8 \text{ cm}$ (3 s.f.) (A1)
 Accept 20.9

[3 marks]

Total [10 marks]

4. (i) (a) $p = 0.159$ (A1)
 $q = 17.5$ (A1)

- (b) (i) $140(9.901)$ (M1)
 $= 1386.14$
 $= 1390 \text{ FFR (3 s.f.)}$ (A1)

OR

$$\frac{140}{0.101} = 1386.14 \quad (M1)$$

$$= 1390 \text{ FFR (3 s.f.)} \quad (A1)$$

Also accept 1387 FFR (you must go up to get 140 GBP)

- (ii) $140(0.024) = 3.36 \text{ GBP Commission}$ (M1)
 $140 - 3.36 = 136.64 \text{ or } 137 \text{ (3 s.f.)}$ (A1)

Note: Commission can be calculated in FFR also for (M1).

- (c) (i) $I = 5000(0.08)(4)$ (M1)
 $= 1600 \text{ FFR}$ (A1)

- (ii) $I = 800(0.06)(4)$
 $= 192 \text{ GBP}$ (M1)
 $192(1.585) = 304.32 \text{ USD or } 304 \text{ USD (3 s.f.)}$ (A1)

- (iii) Paul (A1)

- (iv) Jean: 1600 FFR
 $1600 \div 6.289 = 254.40 \text{ or } 254 \text{ (3 s.f.)}$ (R1)
 Paul: $304.32 \text{ or } 304 \text{ (3 s.f.)}$ (R1)

Note: Accept any same currency comparison.

[13 marks]

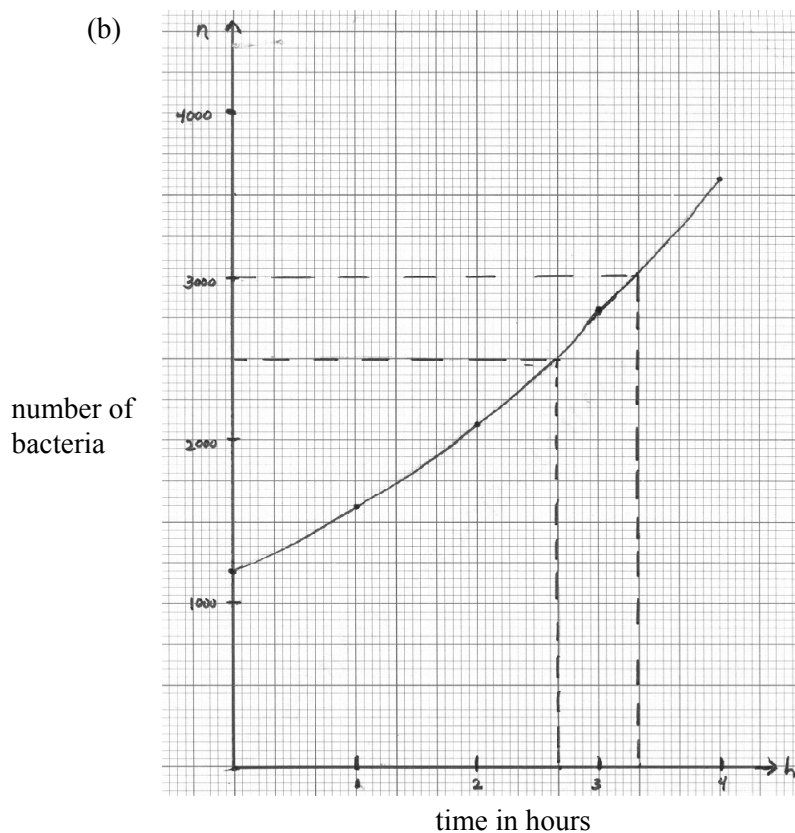
- (ii) Takaya: $1000 + 1000(0.063)(15) = 1945 \text{ JPY}$ (M1)(A1)
 Morimi: $900(1.063)^{15} = 2250 \text{ JPY}$ (M1)(A1)
 Morimi had more (A1)

Note: Award (M0)(A0) for computing interest only for Takaya.
 Award (M1) follow through, (A1) follow through for follow through with only interest for Morimi.
 Award (A1) for “Morimi had more” for comparing the two interests.
 Award (A1) for “Morimi had more” for any reasoning that shows understanding of difference between compound and simple interest.
 Award (A0) for “Morimi had more” after computing interest for Takaya, but interest + principle for Morimi.
 Award (A1) for “Morimi had more” with no work shown at all.

[5 marks]

Total [18 marks]

5.	(i)	(a)	Time in hours (h)	0	1	2	3	4	
			No. of bacteria (n)	1200	1600	2100	2700	3600	(A1)(A1)



Note: Award (A1) for the axes correctly labelled and (A1) for the correct scales.
Award (A2) for 4 or 5 points correctly plotted, (A1) for 2 or 3 correct and (A1) for connecting points with a smooth curve.

- (c) (i) 2500 (M1)(A1)
- (ii) 3hrs 20min (M1)(A1)

Note: Use follow through from graph. If no method is shown from graph give (C1) only for correct answer.

[11 marks]

- (ii) (a) $l = 5 + 2x$ (A1)
- (b) Area of picture plus frame $= (5 + 2x)^2$ (A1)
Therefore $A = (5 + 2x)^2 - 5^2$ (A1)
OR
Area of picture plus frame $= 49 \text{ cm}^2$ (M1)
Since it is a square, length of one side $= 7 \text{ cm}$ (M1)
 $(2x + 5) = 7 \text{ cm}$ (M1)
 $x = 1 \text{ cm}$ (A1)

continued...

Question 5 (ii) continued

(c) $(5 + 2x)^2 - 5^2 = 24$ *(M1)*

$$25 + 20x + 4x^2 - 25 = 24$$

$$4x^2 + 20x - 24 = 0$$
 (M1)

$$x^2 + 5x - 6 = 0$$

$$(x - 1)(x + 6) = 0$$
 (A1)

$$x = 1 \text{ or } x = -6$$

The width is 1 cm *(A1)*

Note: For $4x^2 + 20x - 24 = 0$ correctly solved with no work shown and $x = 1$ give full marks.

[7 marks]

Total [18 marks]

SECTION B

6. (i) (a) (i)

$$T = \begin{matrix} & \begin{matrix} \text{Small} & \text{Medium} & \text{Large} & \text{X Large} \end{matrix} \\ \begin{matrix} \text{Brasil} \\ \text{France} \end{matrix} & \begin{bmatrix} 125 & 125 & 150 & 200 \\ 200 & 200 & 225 & 250 \end{bmatrix} \end{matrix}$$

(A2)

Note: Award (A1) for correct labels, (A1) for correct entries and dimensions.

(ii) Matrix C represents the costs of the different sized T-shirts to the vendor. (A1)

(b) (i) $\begin{bmatrix} 125 & 125 & 150 & 200 \\ 200 & 200 & 225 & 250 \end{bmatrix} \begin{bmatrix} 7 \\ 9 \\ 9 \\ 10 \end{bmatrix} = \begin{bmatrix} 5350 \\ a \end{bmatrix}$ (M2)

$a = 7725$ (A1)

OR

$$a = 200 \times 7 + 200 \times 9 + 225 \times 9 + 250 \times 10 = 7725$$

(M2)
(A1)

Note: Award (M0)(A1) for answer only.

(ii) This represents the total cost to the vendor of the French T-shirts (A1)

(c) $\begin{bmatrix} 50 & 50 & 45 & 50 \\ 25 & 60 & 25 & 40 \end{bmatrix} \begin{bmatrix} 7 \\ 9 \\ 9 \\ 10 \end{bmatrix} = \begin{bmatrix} 1705 \\ 1340 \end{bmatrix}$ (M1)

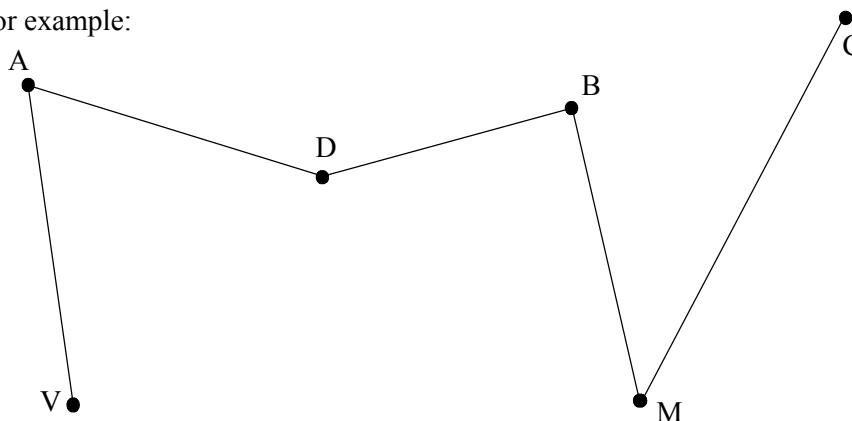
$1705 + 1340 = 3045 \text{ USD}$ (A1)

Note: Award (M1)(A0) if 1705 and 1340 are computed in anyway but no total is given.

[9 marks]

(ii) (a) Any suitable tree; please note, it must have no loops and must be connected.

For example:

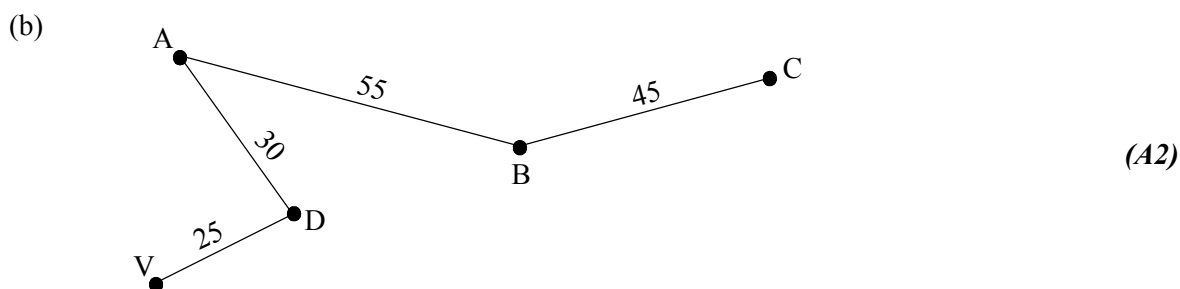


(A2)

Note: Award (A1) for the tree drawn correctly, (A1) for vertices correctly labelled.

continued...

Question 6 (ii) continued



OR

Valley → Abbey → Dodge → Bundy → Clark (A2)

- (c) (i) the vertices represent the towns (A1)
- (ii) the edges represent the roads (A1)
- (d) the degree of Dodge is 4 (A1)
- (e) (i) because not all vertices are of even degree (A1)

Note: Also accept “because it is not Eulenan” and “because there are odd vertices”.

*(ii) $C \rightarrow M \rightarrow V \rightarrow D \rightarrow M \rightarrow B \rightarrow C \rightarrow A \rightarrow B \rightarrow D \rightarrow A \rightarrow V$ (A3)

Note: Award (A1) for starting at either Clark or Valley, (A1) for ending at either Clark or Valley, (A1) for correct paths in between.
 * According to the definition of a path in the IB Mathematical Studies syllabus, drawing a path for this question would not be possible and candidates may so state. However, alternative definitions of a path allow repeated vertices and solutions such as the one listed above would be possible.
 Allow either of these two solutions with no penalty to the candidate.

- (f)
- | | A | C | M | B | V | D |
|---|---|---|---|---|---|---|
| A | 0 | 1 | 0 | 1 | 1 | 1 |
| C | 1 | 0 | 1 | 1 | 0 | 0 |
| M | 0 | 1 | 0 | 1 | 1 | 1 |
| B | 1 | 1 | 1 | 0 | 0 | 1 |
| V | 1 | 0 | 1 | 0 | 0 | 1 |
| D | 1 | 0 | 1 | 1 | 1 | 0 |
- (A3)

Note: Award (A1) for 0's in the diagonal, (A1) for symmetry on either side of diagonal, (A1) for **each entry in the upper triangular section correct**. (2 wrong or more give (A0))

[14 marks]

continued...

Question 6 continued

- (iii) (a) Brad will lose 7 and Janet will gain 7 (accept either or both answers) (A1)

(b)

		Janet			
		D	E	F	Row minimums
	A	-4	6	5	-4 ✓
Brad	B	5	-7	3	-7
	C	-8	0	6	-8
	Column maximum	5 ✓	6	6	

(M1)

(M1)

Note: Award (M1) for row minimum, (M1) for column maximum.

Therefore, the play safe strategy would be Brad rings bell A and Janet rings bell D. (A1)

- (c) (i) Brad should ring bell B (A1)
- (ii) Janet should ring bell D (A1)
- (iii) Brad will win 5 and Janet will lose 5 (accept either or both answers) (A1)

[7 marks]

Total [30 marks]

7. (i) (a) (i) $Z_1 = \frac{765 - 750}{10}$ (M1)

$= 1.5$
 $P(W > 765) = P(Z > 1.5) = 0.0668$ (3 s.f.) (A1)

(ii) $Z_2 = \frac{725 - 750}{10}$ (M1)

$= -2.5$
 $P(W < 725) = P(Z < -2.5) = 0.00621$ (3 s.f.) (A1)

(iii) $P(725 < W < 750) = 0.4332$
 $P(750 < W < 765) = 0.4938$
 $P(725 < W < 765) = 0.4332 + 0.4938$ (M1)
 $= 0.927$ (3 s.f.) (A1)

OR

$1 - (0.0668 + 0.0062) = 0.927$ (3 s.f.) (M1)(A1)

(b) (i) $P(Z > 2) = 0.02275 = 0.0228$ (3 s.f.)
 $P(Z < -2) = 0.02275 = 0.0228$ (3 s.f.)
 $P(\text{reject}) = 2(0.02275) = 0.0455$ (3 s.f.) (M1)
 $= 0.045$ or $2 \times 0.0228 = 0.0456$ (3 s.f.) (A1)
 Also accept 0.0454 (from tables)

(ii) $1000(0.0455) = 45.5$ (M1)
 $= 45$ boxes or 45.6. Accept 45 or 46. (A1)

Note: Award (C2) for all correct answers in part (a) and in (b)(i).

(c) (i) H_0 : The data is normally distributed with mean 750 and standard deviation 10. (C1)
 H_a : The data is not normally distributed with mean 750 and standard deviation 10. (C1)

OR

H_0 : The machine is working properly. (C1)
 H_a : The machine is not working properly. (C1)

(ii)

Values of Z	f_o	f_e	$f_e - f_o$	$(f_e - f_o)^2$
$-1 \leq Z \leq 1$	61	68	7	49
$-2 \leq Z < -1$ or $1 < Z \leq 2$	30	27	-3	9
$Z < -2$ or $Z > 2$	9	5	-4	16

(A6)

Note: Award (A1) for each pair of correct values.

(iii) $\chi^2_{\text{calc}} = \sum \frac{(f_e - f_o)^2}{f_e} = \frac{49}{68} + \frac{9}{27} + \frac{16}{5}$ (M1)(M1)
 $= 4.25$ (3 s.f.) (A1)

continued...

Question 7 continued

- (iv) At 5% level of significance, with 2 degrees of freedom, $\chi^2 = 5.99$ (A1)
 Since $\chi_{\text{calc}}^2 < \chi^2$, we cannot reject H_0 and conclude that the data is (R1)
 normally distributed, that is, that the machine is operating properly. (R1)

Note: Award (R1) for “accept null hypothesis” or “fail to reject null hypothesis”. To award (R2) candidates must write “distribution is normal” or “machine is operating properly”.

[24 marks]

- (ii) (a) $r = \frac{S_{xy}}{S_x S_y}$
 $= \frac{4.16}{(8.96)(0.610)}$ (M1)
 $= 0.76$ (A1)
- (b) There is a fairly strong positive correlation between high school grades and university grades. (A1)(A1)

Note: Award (A1) for strong (or fairly strong) or high, (A1) for positive.

- (c) $y - \bar{y} = \frac{S_{xy}}{S_x^2}(x - \bar{x})$
 $y - 3.04 = \frac{4.16}{8.96^2}(x - 83.5)$ (M1)
 $y = 0.052x - 1.29$ (3 s.f.) (A1)

Note: Award (C2) for correct answer (from calculator).

[6 marks]

Total [30 marks]

8. (a) (i) $v(1) = 1^3 - 4(1)^2 + 4(1)$
 $= 1 \text{ ms}^{-1}$ (A1)

(ii) $v(0.5) = (0.5)^3 - 4(0.5)^2 + 4(0.5)$
 $= 1.125 \text{ ms}^{-1}$ accept 1.13 (3 s.f.) (A1)

[2 marks]

(b) $a = v(1.5) = 1.5^3 - 4(1.5) + 4(1.5)$
 $= 0.375$ (A1)

$b = v(3) = 3^3 - 4(3^2) + 4(3)$
 $= 3$ (A1)

Table (not required)

t	0	0.5	1	1.5	2	2.5	3	3.5	4
v	0	1.125	1	0.375	0	0.625	3	7.875	16

[2 marks]

(c) (i) $\frac{dv}{dt} = 3t^2 - 8t + 4$ (A1)

$3t^2 - 8t + 4 = 0$ (M1)

$(3t - 2)(t - 2) = 0$ (M1)

$t = \frac{2}{3}, t = 2$ (A1)(A1)

(ii) The function is changing from acceleration to deceleration
 or velocity changes from increasing to decreasing
 or kite is stationary or velocity is zero (R1)(R1)

Note: Award (R1) for acceleration, (R1) for deceleration.

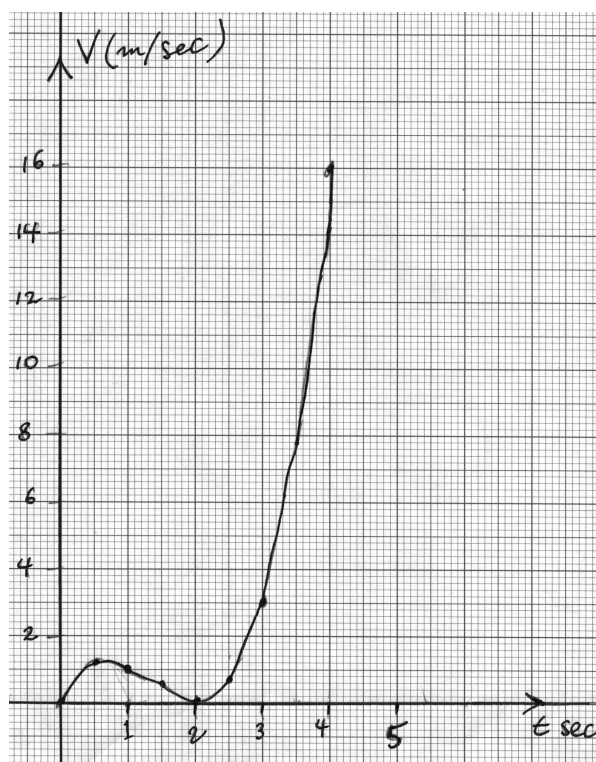
Gradient = 0 (A1)

[8 marks]

continued...

Question 8 continued

(d)



(A5)

Note: Award (A1) for axes correctly labelled, (A1) if scales correct, (A1) for correct general shape of curve, (A1) for each turning point in approximately the correct place.

[5 marks]

time t	motion
$t = 0$	stopped
$0 < t < \frac{2}{3}$	accelerating (increasing in velocity)
$t = \frac{2}{3}$	stopped accelerating
$\frac{2}{3} < t < 2$	decelerating (decreasing in velocity)
$t = 2$	stopped decelerating
$2 < t \leq 4$	accelerating

(A1)

(A1)

(A1)

Note: Stops may be left out

[3 marks]

(f) $u(t) = 2t^2 - 3t + c$

(M1)

$$3 = 2(1)^2 - 3(1) + c$$

(M1)

$$3 = -1 + c$$

$$4 = c$$

(A1)

$$u(t) = 2t^2 - 3t + 4$$

(A1)

[4 marks]

continued...

Question 8 continued

(g) $4t - 3 = 0$ (M1)

$t = \frac{3}{4}$ i.e. after $\frac{3}{4}$ second (A1)

[2 marks]

(h) $a\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right) - 3$ (M1)

$= 2 - 3 = -1$
decelerating (A1)

Note: Accept alternative correct explanations.

[2 marks]

(i) Second kite accelerates from

$t = \frac{3}{4}$ (minimum value) (or any similar reasoning) (M1)

Therefore, after $t = 2$ (from first curve) both are accelerating between $t = 2$ and $t = 4$ (A1)

Note: Accept “after two seconds”.

[2 marks]

Total [30 marks]